2023

MATHEMATICS — HONOURS

Paper : CC-10

(Mechanics)

Full Marks : 65

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

1. Answer the following multiple choice questions with only one correct option. Choose the correct option with proper justification : (1+1)×10

(a) If X, Y be the algebraic sum of the resolved parts of a system of coplanar forces along the rectangular axes OX and OY respectively and G be the algebraic sum of the resolved parts of the moments the forces about O, then the locus of the point about which the algebraic sum of the moments of the forces is equal to a constant G', is given by :

(i) $Yx - Xy = G' - G$ (ii) $Yx - Xy = G - G'$

(iii) $Yy - Xx = G' - G$ (iv) $Yy - Xx = G - G'$.

(b) Suppose a force F is acting along the line $\frac{x-x_1}{l} = \frac{y-y_1}{m} = \frac{z-z_1}{n}$, then the moment of F about z-axis will be

(i) $F(x_1 m - y_1 l)$ (ii) $F(y_1 n - z_1 m)$

(iii) $F(z_1 l - x_1 n)$ (iv) $F(z_1 l - y_1 n)$.

(c) A lamina in the form of a cycloid rests on the top of another cycloidal lamina. The radii of the generating circles of the upper and lower cycloid are 'a' and 'b' respectively. If 'h' is the height of the c.g. of the upper cycloid above its vertex, then the equilibrium will be stable if :

(i) $h > \frac{2ab}{a+b}$ (ii) $h < \frac{2ab}{a+b}$

(iii) $h > \frac{4ab}{a+b}$ (iv) $h < \frac{4ab}{a+b}$.

(d) If the resultant R of two forces $2P$ and P is the geometric mean between the greatest and the least values of R, then the angle between the forces P and $2P$ is

(i) $\frac{\pi}{2}$ (ii) $\frac{\pi}{6}$

(iii) $\frac{2\pi}{3}$ (iv) $\frac{5\pi}{6}$.

(e) The potential energy U of a conservative system varies with distance x and $U = 3x^2 - 2x$. Then the force acting on the system

(i) is proportional to x (ii) is proportional to x^2

(iii) decreases constantly (iv) decreases linearly with x.

(f) A man starts walk at 4 miles per hour and his speed at any instant is inversely proportional to $(x + 10)$, where x is the number of miles he has walked. Then the time he takes to walk 20 miles is

(i) 10 hours (ii) 15 hours

(iii) 20 hours (iv) None of these.

(g) If the radial velocity is proportional to the transverse velocity of a particle moving in a plane curve, then the plane curve is

(i) a parabola

(ii) a hyperbola

(iii) a cardioid

(iv) an equiangular spiral.

(h) A particle describes the path $p^2 = ar$ under a force F to the pole. Then the law of force will be

(i) $F \propto \frac{1}{r}$ (ii) $F \propto \frac{1}{\sqrt{r}}$

(iii) $F \propto \frac{1}{r^2}$ (iv) $F \propto r^2$.

(i) If the ratio of the elliptical orbits of two planets is $\frac{9}{16}$, then the ratio of their periodic time is

(i) $\frac{3}{4}$ (ii) $\frac{4}{9}$

(iii) $\frac{8}{27}$ (iv) $\frac{27}{64}$.

(j) A uniform hollow cylinder (circular) is rolling down a rough plane inclined at an angle α to the horizon. Then the kinetic energy of the cylinder at any time t is given by

(i) Mv^2 (ii) $\frac{1}{2}Mv^2$

(iii) $\frac{1}{3}Mv^2$ (iv) $\frac{1}{4}Mv^2$,

where M is the mass of the cylinder and v is the velocity of its centre of mass at any time t.

Unit - 1

(Marks : 05)

2. Answer *any one* question :

(a) Two uniform similar rods of same materials PQ and QT of length $2l$ and $2l'$ respectively are rigidly united at Q and suspended freely from P. If they rest inclined at an angles α and β respectively to the vertical, prove that $(l^2 + 2ll')\sin\alpha = l'^2 \sin\beta$. 5

(b) OA, OB, OC are three co-terminous edges of a cube and AA', BB', CC' and OO' are diagonals. Along BC', CA', AB' and OO' act forces equal to X, Y, Z and R respectively. Show that they are equivalent to a single resultant if $(YZ + ZX + XY)\sqrt{3} + R(X + Y + Z) = 0$. 5

Unit - 2

(Marks : 05)

3. Answer *any one* question :

(a) Four equal rods each of weight W form a rhombus $ABCD$ with smooth hinges at the joints. The frame is suspended by the end A and a weight W' is attached at C. A stiffening rod of negligible weight joins the middle points of AB, AD keeping these inclined at an angle α to AC. Show that the thrust in the stiffening rod is $(4W + 2W')\tan\alpha$. 5

(b) A beam of length *'l'* rests with its ends on two smooth planes which intersect in a horizontal line. If the inclination of the planes to the horizon are α and β and the c.g. of the beam divides it in the ratio $a : b$; find the position of equilibrium of the beam and show that it is unstable. 5

Unit - 3

(Marks : 10)

4. Answer *any two* questions :

(a) Deduce the radial and cross-radial components of velocity and acceleration of a particle moving in a plane. 5

(b) A shot of mass *'m'* penetrates a thickness *'s'* into a fixed beam of mass M. Prove that if M is free to move, the thickness penetrated is $\frac{Ms}{M+m}$. 5

(c) If a particle moves from one position to another position on a smooth plane curve under the action of conservative forces in the plane, then show that the change in kinetic energy is equal to the work done by the forces. 5

(d) An engine is pulling a train starting from rest, works at a constant rate H units per second. If M is the mass of the train and the frictional resistance is $K \times (\text{velocity})^2$, find the distance when the train will obtain velocity u. Also find the maximum speed of the train. 5

Please Turn Over

Unit - 4

(Marks : 15)

5. Answer ***any one*** question :

(a) A particle is thrown upwards with speed V. If the air resistance is assumed to vary as the square of the speed and the resistance is equal to gravity (supposed constant) when the speed is U, show that the particle will rise for a time $\frac{U}{g}\tan^{-1}\left(\frac{V}{U}\right)$. Also prove that the velocity with which it returns to the point of projection is $\frac{UV}{\left(U^2+V^2\right)^{\frac{1}{2}}}$. 4+3

(b) A point is moving in a straight line with an acceleration μx towards a fixed centre in the straight line and with an additional acceleration $L\cos pt$, where μ, L and p are positive constants and x is the distance from the fixed point. If $p=\sqrt{\mu}$ and $x=a, \frac{dx}{dt}=0$, when $t=0$, find x and $\frac{dx}{dt}$ at any time t. 7

6. Answer ***any one*** question :

(a) A particle moves under a central acceleration $\frac{\mu}{r^3}$. It is projected from an apse at a distance 'a' from the centre of force with a velocity equal to $\sqrt{2}$ times the velocity in a circle at the same distance. Prove that the path is $r\cos\left(\frac{\theta}{\sqrt{2}}\right)=a$. 8

(b) A particle projected along the inner surface of a rough sphere is acted on by no forces. Show that it will return to the point of projection at the end of time $\frac{a}{\mu v}\left(e^{2\pi\mu}-1\right)$, where '$a$' is the radius of the sphere, 'v' is the velocity of projection and 'μ' is the coefficient of friction. 8

Unit - 5

(Marks : 10)

7. Answer ***any one*** question :

(a) (i) If two smooth spheres of masses m_1 and m_2 moving with respective velocities u_1 and u_2 in the same direction impinge directly, find (i) their velocities after impact and (ii) loss of K.E.

(ii) A ball is thrown from a point on a smooth, horizontal ground with a velocity V at an angle α to the horizon. Show that the total time for which the ball rebounds on the ground is $\frac{2V\sin\alpha}{g(1-e)}$ and that its distance from the starting point when it ceases to rebound is $\frac{V^2\sin 2\alpha}{g(1-e)}$, e being the coefficient of elasticity between the ball and the ground. 5+5

(b) (i) Define linear momentum for a multiparticle system consisting of particles with masses $m_1, m_2,...,m_N$ and velocities $\vec{v}_1, \vec{v}_2,...,\vec{v}_N$. Interpret it in terms of motion of centre of mass of the system.

(ii) Write down linear momentum principle for such multiparticle system and interpret it in terms of an isolated system.

(iii) The position of a particle P of mass 'm' at time t is given by $x = at$, $y = 2at$, $z = 3at$. Find angular momentum of P about a point $A(a, a, a)$ and about the axis of z at any time t. (1+1)+(2+2)+(2+2)